

# 511 keV line and diffuse gamma rays from moduli

Shinta Kasuya<sup>a</sup> and Masahiro Kawasaki<sup>b</sup>

<sup>a</sup> Department of Information Science, Kanagawa University, Kanagawa 259-1293, Japan

<sup>b</sup> Institute for Cosmic Ray Research, University of Tokyo, Chiba 277-8582, Japan

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We obtain the spectrum of gamma ray emissions from the moduli whose decay into  $e^+e^-$  accounts for the 511 keV line observed by SPI/INTERGRAL. The moduli emit gamma rays through internal bremsstrahlung, and also decay directly into two gammas via tree and/or one-loop diagrams. We show that the internal bremsstrahlung constrains the mass of the moduli below  $\sim 40$  MeV model-independently. On the other hand, the flux of two gammas directly decayed from the moduli through one loop diagrams will exceed the observed galactic diffuse gamma-ray background if the moduli mass exceeds  $\sim 20$  MeV in the typical situation. Moreover, forthcoming analysis of SPI data in the range of 1 – 8 MeV may detect the line emission with the energy half the moduli mass in the near future, which confirms the decaying moduli scenario.

## I. INTRODUCTION

The spectrometer SPI on the International Gamma-Ray Astrophysics Laboratory (INTEGRAL) confirmed with its high resolution that 511 keV line gamma-rays are coming from the Galactic bulge [1]. It is explained by the annihilation of positrons with electrons, but the source of the positrons is difficult to be traditional astrophysical objects, since the emission region observed is wide-spread. Thus, it may be plausible that the particle dark matter annihilating or decaying into positrons could explain the observed 511 keV line [2, 3, 4].

Among them, the moduli may be the simplest candidate, since it appears very naturally in supersymmetric and/or superstring models [3]. One of the prominent feature of the moduli is that they can decay into two gammas directly via tree or one loop diagrams. This could be a signal for proving the scenario once one would observe the line emission with the energy half of their mass, i.e.,  $m_\phi/2$ , where  $m_\phi$  is the moduli mass.

In this article, we consider the photon emission from the moduli through the internal bremsstrahlung [5], and direct productions via tree or one-loop diagrams in our galaxy, as well as from extragalactic emission. We evaluate these line and/or diffuse gamma-ray spectra, assuming that the flux of the 511 keV line in the galactic center, observed by SPI/INTEGRAL, is explained by the positron production by the moduli decay. Comparing them with the diffuse gamma-ray background observations from INTEGRAL, COMPTEL, EGRET, and SMM, we obtain the constraints on the moduli mass.

## II. 511 KEV LINE

We consider the moduli  $\phi$  couple to electrons by the following Yukawa coupling [3]:

$$\mathcal{L} = \frac{m_e}{M_P} \phi \bar{e} e, \quad (1)$$

where  $m_e$  is the electron mass, and  $M_P \simeq 2.4 \times 10^{18}$  GeV is the (reduced) Planck mass. The decay width is given

by

$$\Gamma_e \equiv \Gamma(\phi \rightarrow e^+ + e^-) \simeq \frac{1}{8\pi} \left( \frac{m_e}{M_P} \right)^2 m_\phi. \quad (2)$$

In order to explain the 511 keV line gamma-rays from the galactic center with the flux of  $\Phi_{511} \simeq 10^{-3} \text{ cm}^{-2} \text{ sec}^{-1}$  observed by SPI/INTEGRAL [1], the amount of the moduli should be [2, 3]

$$\Omega_\phi \simeq \left( \frac{\Phi_{511}}{10^{-3} \text{ cm}^{-2} \text{ sec}^{-1}} \right) \left( \frac{\tau_\phi}{10^{27} \text{ sec}} \right) \left( \frac{m_\phi}{\text{MeV}} \right), \quad (3)$$

where  $\Omega_\phi$  and  $\tau_\phi$  is the density parameter and the lifetime of the moduli, respectively, and we apply a spherically symmetric profile for the dark matter density with  $\rho \propto r^{-1.2}$ . Taking  $\tau_\phi = \Gamma_e^{-1}$ , the abundance should thus be  $\Omega_\phi \simeq 4 \times 10^{-4}$ .<sup>†</sup>

## III. INTERNAL BREMSSTAHUNG

Let us start with investigating the internal bremsstrahlung spectrum. In the context of annihilating dark matter, the spectrum of the gamma-ray due to the internal bremsstrahlung has been obtained in Ref. [5], and it is found that the energy of the emitted photons should be smaller than  $\sim 20$  MeV. Here we consider analogous process in the context of decaying particles. The tree diagram of the decay process is shown in Fig. 1(a). The internal bremsstrahlung photons are emitted as in Fig.1(b). The decay rate for the internal bremsstrahlung is written as (See the Appendix)

$$\frac{d\Gamma_{br}}{d\varepsilon_\gamma} = \frac{y^2 e^2}{(2\pi)^3}$$

<sup>†</sup> Moduli could contribute considerably to dark matter, i.e.,  $\Omega_\phi \sim \Omega_{DM} \sim 0.3$ , provided that the coupling of the moduli to electrons is  $\mathcal{L} = s \frac{m_e}{M_P} \phi \bar{e} e$  with  $s \sim 0.03$ . Notice that this change does not alter the constraints of internal bremsstrahlung or decay into two photons discussed in the following sections.

$$\times \left[ \left( \frac{\varepsilon_\gamma}{m_\phi} - 1 + \frac{m_\phi}{2\varepsilon_\gamma} - \frac{2m_e^2}{m_\phi\varepsilon_\gamma} + \frac{4m_e^2}{m_\phi^2} \right) \log \frac{1+\eta}{1-\eta} + \left( \frac{m_e^2}{m_\phi\varepsilon_\gamma} + \frac{2m_e^4}{m_\phi^3\varepsilon_\gamma} \right) \frac{2\eta}{1-\eta^2} \right], \quad (4)$$

where  $y = m_e/M_P$ ,  $\varepsilon_\gamma$  is the photon energy and

$$\eta = \left( 1 - \frac{4m_e^2}{m_\phi(m_\phi - 2\varepsilon_\gamma)} \right)^{1/2}. \quad (5)$$

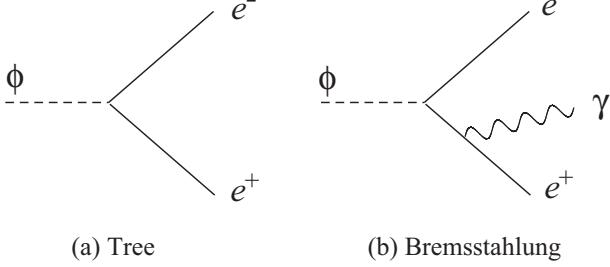


FIG. 1: Diagrams of decay of the moduli into positron and electron pair with (b) and without (a) internal bremsstahlung photon.

Since the (tree) decay rate into positron and electron pair is given by

$$\Gamma_e = \frac{y^2 m_\phi}{8\pi} \left( 1 - \frac{4m_e^2}{m_\phi^2} \right)^{3/2}, \quad (6)$$

the flux spectrum for the internal bremsstahlung, normalized to the observed 511 keV line flux, becomes [5]

$$\begin{aligned} \frac{d\Phi_{br}}{d\varepsilon_\gamma} &\simeq \frac{1}{2} \Phi_{511} \frac{1}{\Delta\Omega} \left[ \frac{f}{4} + (1-f) \right]^{-1} \frac{1}{2} \frac{1}{\Gamma_e} \frac{d\Gamma_{br}}{d\varepsilon_\gamma} \\ &\simeq \frac{1}{4} \Phi_{511} \frac{1}{\Delta\Omega} \left[ \frac{f}{4} + (1-f) \right]^{-1} \\ &\times \frac{4\alpha_{em}}{\pi m_\phi} \left( 1 - \frac{4m_e^2}{m_\phi^2} \right)^{-3/2} \\ &\times \left[ \left( \frac{\varepsilon_\gamma}{m_\phi} - 1 + \frac{m_\phi}{2\varepsilon_\gamma} - \frac{2m_e^2}{m_\phi\varepsilon_\gamma} + \frac{4m_e^2}{m_\phi^2} \right) \log \frac{1+\eta}{1-\eta} \right. \\ &\quad \left. + \left( \frac{m_e^2}{m_\phi\varepsilon_\gamma} + \frac{2m_e^4}{m_\phi^3\varepsilon_\gamma} \right) \frac{2\eta}{1-\eta^2} \right]. \end{aligned} \quad (7)$$

Here a half of the 511 keV flux is emitted from an angular region  $\Delta\Omega$  given by the Gaussian FWHM of  $8^\circ$ , and  $f$  is the fraction of the positron decay via positronium formation over all the incident positrons [6].

We plot the spectrum for  $m_\phi = 10 - 60$  MeV in Fig. 2. The dotted lines represent the uncertainty coming from the observational uncertainties of 511 keV flux of  $\Phi_{511} = (1.07 \pm 0.03) \times 10^{-3} \text{ cm}^{-2} \text{ sec}^{-1}$  and  $f = 0.967 \pm 0.022$  [1].

We also display the COMPTEL and EGRET data [7]. In order not to exceed the observed flux, the mass of the moduli should be  $m_\phi \lesssim 40$  MeV. Notice that this constraint is model independent, since no coupling constant  $y$  appears in the formula of the spectrum.

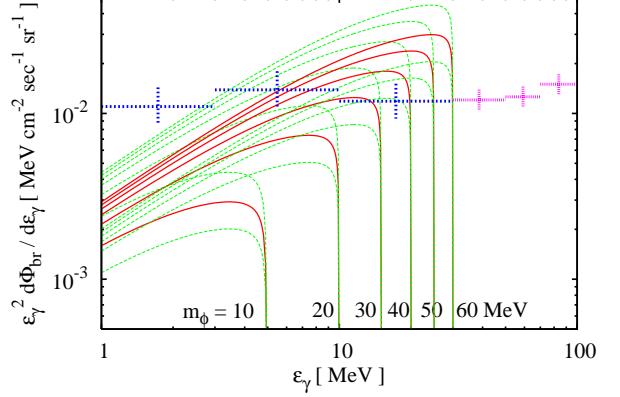


FIG. 2: Internal bremsstahlung spectrum for the various moduli masses. Dotted lines show the uncertainty bands. Also plotted are the COMPTEL ( $\varepsilon_\gamma < 30$  MeV) and EGRET data ( $\varepsilon_\gamma > 30$  MeV) of the galactic diffuse background.

#### IV. DECAY INTO TWO PHOTONS

The moduli may decay into two photons through non-renormalizable interactions suppressed by the gravitational strength. When the moduli are the dilation-type, they may decay via the following interaction [8]:

$$\mathcal{L} = \frac{b}{4M_P} \phi F_{\mu\nu} F^{\mu\nu}, \quad (8)$$

where  $b$  is of the order  $O(1)$  parameter, which depends on the type of superstring theory and compactification. Then the decay width via this interaction is given by

$$\Gamma_{2\gamma}^{\text{tree}} = \frac{b^2}{64\pi} \frac{m_\phi^3}{M_P^2}. \quad (9)$$

Even the above interaction is absent, the moduli can decay into two photons through one loop diagrams, since, at least, the moduli couple to positrons and electrons via Yukawa interaction (1). In this case, the decay width is estimated as

$$\Gamma_{2\gamma}^{\text{loop}} \simeq \frac{N^2}{4\pi} \left( \frac{\alpha_{em}}{4\pi} \right)^2 \frac{m_\phi^3}{M_P^2}. \quad (10)$$

Here we include  $N$ , the number of the particle species running the loop, in the formula. Since the moduli have to couple to electrons (positrons), it is likely to couple to other leptons, such as muons and taus. Therefore, it may typically be  $N = 3$ , but larger  $N$  could be possible as well.

Let us first estimate the extragalactic diffuse gamma ray flux by the moduli decay. Since the velocity dispersion of the moduli is negligible, two photons emitted have monochromatic energy of  $m_\phi/2$  when they are produced. Taking into account the dilution by the cosmic expansion, the photon flux can be written as [8]

$$\frac{d\Phi_{\text{dif}}}{d\varepsilon_\gamma} = \frac{2n_{\phi,0}\Gamma_{2\gamma}}{4\pi H_0} \left(\frac{\varepsilon_\gamma}{m_\phi}\right)^{3/2} F(m_\phi/2\varepsilon_\gamma) \times \exp\left[-\int_0^{m_\phi/2\varepsilon_\gamma} dx \frac{\Gamma_{2\gamma}}{H_0} x^{-5/2} F(x)\right], \quad (11)$$

where  $n_{\phi,0}$  is the present number density of moduli,  $H_0 = 100h$  km/sec/Mpc is the present Hubble parameter and  $F(x) = [\Omega_m + (1 - \Omega_m - \Omega_\Lambda)/x + \Omega_\Lambda/x^3]^{-1/2}$ . Here we use  $\Omega_m = 0.27$ ,  $\Omega_\Lambda = 0.73$ , and  $h = 0.71$  [9]. Since  $\Gamma_{2\gamma}^{-1} \gg H_0^{-1}$ , Eq.(11) can be simplified to

$$\frac{d\Phi_{\text{dif}}}{d\varepsilon_\gamma} = \frac{2\Omega_\phi \rho_{C,0} \Gamma_{2\gamma}}{4\pi m_\phi H_0} \left(\frac{\varepsilon_\gamma}{m_\phi}\right)^{3/2} F(m_\phi/2\varepsilon_\gamma). \quad (12)$$

We plot the extragalactic diffuse gamma-ray spectra of three typical decay rates for various moduli masses in Fig.3. The observational constraints are from COMPTEL (below 30 MeV) [10] and EGRET (above 30 MeV) data [11]. We also plot the data from the Solar Maximum Mission (SMM) Gamma-Ray Spectrometer in the energy range 0.3 – 8.0 MeV [12]. For the direct decay rate, the mass of the moduli is very restricted such that  $m_\phi \lesssim 1.5$  MeV. On the other hand, when two photons are emitted through loop diagrams, the constraint on the mass is relatively less stringent:  $m_\phi \lesssim 30$  (60) MeV for  $N = 3$  (1).

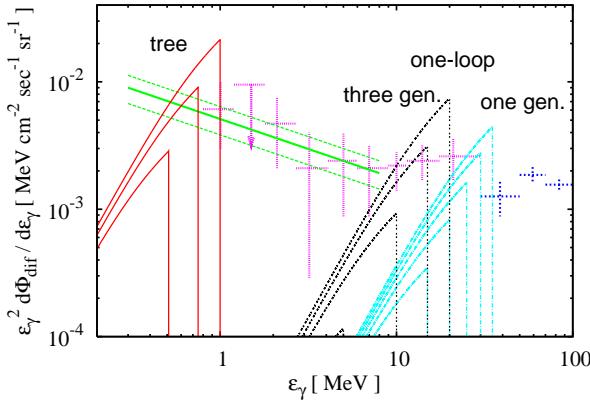


FIG. 3: Extragalactic diffuse gamma spectrum. We show  $m_\phi = 1.022, 1.5, \text{ and } 2$  MeV for the direct decay rate (9),  $m_\phi = 10, 20, 30, \text{ and } 40$  MeV for the one-loop decay with  $N = 3$ , and  $m_\phi = 30, 40, 50, 60, \text{ and } 70$  MeV for  $N = 1$ . Crosses are from COMPTEL below 30 MeV, and EGRET data above 30 MeV. We also plot SMM data with thin solid line with two dashed lines above and below, representing 25% uncertainties.

The most stringent constraints will come from the two photon emission from our galaxy. Since the cosmological redshift is negligible, it results in a line spectrum with energy half the moduli mass,  $m_\phi/2$ . Since our assumption is that the annihilations of the positron from the moduli decay account for the observed 511 keV line in the galactic bulge, the ratio of the flux of 511 keV line and two-gamma directly decayed from the moduli can be expressed as

$$\frac{\Phi_{2\gamma}}{\Phi_{511}} \simeq \frac{\Gamma_{2\gamma}}{\Gamma_e} \left[ \frac{f}{4} + (1-f) \right]^{-1}. \quad (13)$$

Thus, the spectrum of the line gamma is given by

$$\varepsilon_\gamma^2 \frac{d\Phi_{2\gamma}}{d\varepsilon_\gamma} \simeq \frac{r\Phi_{2\gamma}}{\Delta\Omega} \left( \frac{\varepsilon_\gamma}{\Delta\varepsilon_\gamma} \right) \varepsilon_\gamma, \quad (14)$$

where  $\Delta\Omega$  is the solid angle for  $330^\circ < \ell < 30^\circ$  and  $|b| < 10^\circ$ , and  $r \approx 0.993$  is the fraction of the flux coming from the corresponding solid angle.  $\Delta\varepsilon_\gamma/\varepsilon_\gamma$  is the fractional energy resolution of the observation. We adopt  $\Delta\varepsilon_\gamma/\varepsilon_\gamma \simeq 0.08$  for  $\varepsilon_\gamma = 1-30$  MeV for COMPTEL, while  $\Delta\varepsilon_\gamma/\varepsilon_\gamma \simeq 0.13$  for  $\varepsilon_\gamma = 30-100$  MeV for EGRET. For INTEGRAL, the energy resolution is  $\Delta\varepsilon_\gamma \simeq 2$  keV for  $\varepsilon_\gamma = 0.5-1$  MeV.

Let us first examine the case which the moduli decay into two photons by the tree decay width (9). Then the ratio of the flux becomes

$$\frac{\Phi_{2\gamma}}{\Phi_{511}} \simeq \frac{b^2}{8} \left( \frac{m_\phi}{m_e} \right)^2 \left[ \frac{f}{4} + (1-f) \right]^{-1}. \quad (15)$$

Since  $[f/4 + (1-f)]^{-1} \simeq 3.6$ , the two-gamma flux is much larger than 511 keV line, which is completely excluded by observations. For  $m_\phi = 2m_e$ , both of the component could explain the 511 keV line, if we lower the moduli abundance. In this case, however, continuum spectrum just below 511 keV cannot be explained by  $3\gamma$  continuum flux from positronium formation, which was the confirmation of the 511 keV line by positron annihilation [13].

For the decay via one-loop diagrams, the allowed range of the moduli mass is  $m_\phi \lesssim 40$  MeV for  $N = 1$ , as shown in Fig. 4. COMPTEL and EGRET data are taken from Ref. [7], while INTEGRAL data is from Ref. [13]. Diffuse spectrum above 1 MeV observed by INTEGRAL has not been analyzed yet, but the fractional energy resolution is about 0.1% for  $\varepsilon_\gamma = 1-8$  MeV [14]. We thus also show the line spectrum with energy resolution of 0.1% below 8 MeV. If the new analysis of INTEGRAL in the range of  $\varepsilon_\gamma = 1-8$  MeV would be done, the corresponding constraint on the moduli mass will become  $m_\phi \lesssim 10$  MeV if INTEGRAL detects the similar flux obtained by COMPTEL.

As discussed above, the moduli is likely to couple to other leptons, if it has a coupling to electrons (positrons) via Yukawa coupling (1). We show the emission line spectrum for the case that the moduli couple to all three generation of leptons in Fig. 5. Current COMPTEL data

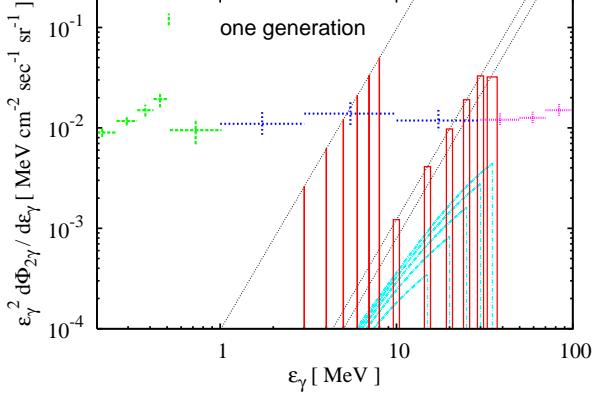


FIG. 4: Line spectra of two photons decayed via one-loop diagram with  $N = 1$  from the moduli in our galaxy. Three dotted lines show the fractional energy resolution of 0.13, 0.08, and 0.001, from the bottom to the top. Data are from INTEGRAL ( $\varepsilon_\gamma < 1$  MeV) [13], COMPTEL ( $\varepsilon_\gamma = 1 - 30$  MeV), and EGRET ( $\varepsilon_\gamma > 30$  MeV) [7]. Extragalactic diffuse emission spectra (12) are also shown for comparison.

restricts the moduli mass as  $m_\phi \lesssim 20$  MeV. The future analysis of INTEGRAL data of  $\varepsilon_\gamma = 1 - 8$  MeV will exclude the moduli mass above 6 MeV provided that the observed diffuse flux is as large as that of COMPTEL. Moreover, the emission line could even be detected for  $m_\phi \lesssim 6$  MeV ( $\varepsilon_\gamma \lesssim 3$  MeV). Notice that the constraint  $\varepsilon_\gamma \lesssim 3$  MeV is recently claimed in Ref. [15], which considered the inflight bremsstahlung.

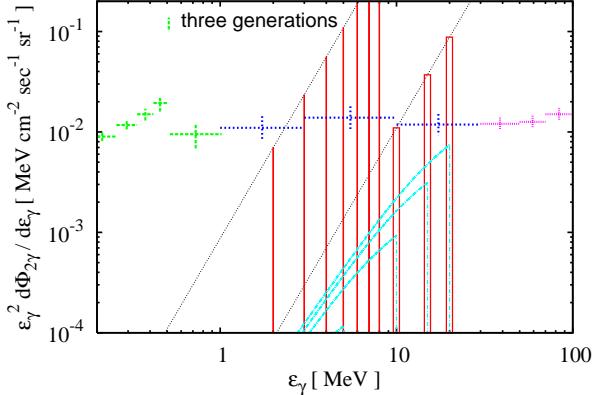


FIG. 5: Line spectra of two photons decayed via one-loop diagrams with  $N = 3$  from the moduli in our galaxy. Dotted lines show the fractional energy resolution of 0.08 (down) and 0.001 (up). Data are the same as in Fig. 4.

## V. CONCLUSIONS

We have considered various gamma-ray emissions from the decay of the moduli when it is assumed that the

annihilation of the positron, into which the moduli decay, provides the 511 keV line emission observed by SPI/INTEGRAL. The flux of the photons emitted via internal bremsstahlung puts the limit on the moduli mass to be  $m_\phi \lesssim 40$  MeV. This is model independent constraint.

On the other hand, the moduli can decay into two photons by tree and/or one-loop diagrams, which gives more stringent constraint on the moduli mass. We have used extragalactic and galactic diffuse gamma-ray background observation to put the upper limit on the moduli mass. As for the extragalactic background emission, the constraints are relatively loose. For instance, the decay width of one-loop diagrams lead to  $m_\phi \lesssim 60$  (30) MeV for one (three) generation(s) of leptons running the loop. If the moduli can decay into two photons via tree coupling, the allowed mass of the moduli is very narrow:  $(1.022 \text{ MeV} \leq) m_\phi \lesssim 1.5$  MeV.

Galactic diffuse gamma-ray spectrum puts the most stringent limit. If the moduli have tree level interactions with photons with of  $O(1)$  coupling, too much photons are emitted once the flux is normalized to that of the 511 keV line of the positron annihilations from the moduli decay. Since there is a coupling of the moduli with electrons (positrons), the moduli can decay into two photons via one-loop diagrams. We thus obtained the mass limit of  $m_\phi \lesssim 40$  MeV for the electron-positron running loop diagram.

It is likely to have stronger coupling of the moduli with photons, since there is no symmetry to restrict the moduli to couple only to the first generation of lepton. We thus have the allowed mass range as  $m_\phi \lesssim 20$  MeV currently, if all three generations of leptons run the loop. Moreover, if the future analysis of the INTEGRAL data above 1 MeV is done, we may even detect the line emission from the moduli due to its high energy resolution of  $\Delta\varepsilon_\gamma/\varepsilon_\gamma \simeq 0.001$ , which may become the confirmation of decaying moduli scenario.

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## APPENDIX A: INTERNAL BREMSSTAHLUNG DECAY RATE

Here we derive the decay rate of the internal bremsstahlung. The tree Yukawa interaction has the form,  $\mathcal{L} = y\phi e\bar{e}$ . There are two diagrams contributing to this process: the photon is emitted either electron or positron external lines. Let us consider the process in the rest frame of the moduli, whose momentum is

$q = (m_\phi, \mathbf{0})$ . Let the momenta of electron, positron, and the emitted photon be  $p_1 = (E_1, \mathbf{p}_1)$ ,  $p_2 = (E_2, \mathbf{p}_2)$ , and  $k$  with  $|k| = \varepsilon_\gamma$ , respectively. Then the total invariant amplitude is calculated as

$$\begin{aligned}
|\mathcal{M}|^2 &= \sum_{\text{spin}, \epsilon} |\mathcal{M}_1 + \mathcal{M}_2|^2 \\
&= 4y^2 e^2 \left[ \frac{(p_2 \cdot k)}{(p_1 \cdot k)} + \frac{(p_1 \cdot k)}{(p_2 \cdot k)} \right. \\
&\quad + \frac{2}{(p_1 \cdot k)(p_2 \cdot k)} \\
&\quad \times \{(p_1 \cdot p_2) + (p_1 \cdot k)\} \{(p_1 \cdot p_2) + (p_2 \cdot k)\} \\
&\quad - \frac{m_e^2}{(p_1 \cdot k)^2} \{(p_1 \cdot p_2) - (p_1 \cdot k) + (p_2 \cdot k)\} \\
&\quad + \frac{3m_e^2}{(p_2 \cdot k)^2} \{(p_1 \cdot p_2) + (p_1 \cdot k) + (p_2 \cdot k)\} \\
&\quad + \frac{2m_e^2}{(p_1 \cdot k)} + \frac{m_e^4}{(p_1 \cdot k)^2} + \frac{3m_e^4}{(p_2 \cdot k)^2} - \frac{2m_e^4}{(p_1 \cdot k)(p_2 \cdot k)} \left. \right] \\
&= 4y^2 e^2 \left[ \left( \varepsilon_\gamma - m_\phi + \frac{m_\phi^2}{2\varepsilon_\gamma} - \frac{2m_e^2}{\varepsilon_\gamma} + \frac{2m_e^2}{m_\phi} \right) \right. \\
&\quad \times \left\{ \frac{1}{E_1 - (\frac{m_\phi}{2} - \varepsilon_\gamma)} + \frac{1}{\frac{m_\phi}{2} - E_1} \right\} \\
&\quad + \frac{-\frac{m_e^2}{2} + \frac{2m_e^4}{m_\phi^2}}{\{E_1 - (\frac{m_\phi}{2} - \varepsilon_\gamma)\}^2} + \frac{\frac{4m_e^2}{m_\phi}}{E_1 - (\frac{m_\phi}{2} - \varepsilon_\gamma)} \\
&\quad \left. + \frac{\frac{3m_e^2}{2}}{(\frac{m_\phi}{2} - E_1)^2} \right], \tag{A1}
\end{aligned}$$

where  $\mathcal{M}_1$  and  $\mathcal{M}_2$  denote the invariant amplitude for each contributed diagrams. Then, the decay rate is expressed as

$$\begin{aligned}
\frac{d\Gamma_{br}}{d\varepsilon_\gamma} &= \frac{1}{2m_\phi} \int \frac{4\pi\varepsilon_\gamma^2}{(2\pi)^3 2\varepsilon_\gamma} \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \\
&\quad \times (2\pi)^4 \delta^{(4)}(q - p_1 - p_2 - k) |\mathcal{M}|^2
\end{aligned}$$

$$= \frac{1}{8(2\pi)^3 m_\phi} \frac{\varepsilon_\gamma}{m_\phi} \int dE_1 \overline{|\mathcal{M}|^2}. \tag{A2}$$

The range of the integration over  $E_1$  is estimated as follows. Momentum conservation leads to  $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k} = \mathbf{0}$ , which recasted as  $|\mathbf{p}_2|^2 = |\mathbf{p}_1 + \mathbf{k}|^2 = |\mathbf{p}_1|^2 + |\mathbf{k}|^2 + 2|\mathbf{p}_1|\varepsilon_\gamma \cos\theta$ , where  $\theta$  is the angle between  $\mathbf{p}_1$  and  $\mathbf{k}$ . Since  $|\cos\theta| \leq 1$ , the allowed range is  $E_- \leq E_1 \leq E_+$ , where

$$E_\pm = \frac{1}{2} \left[ m_\phi - \varepsilon_\gamma \pm \varepsilon_\gamma \left\{ 1 - \frac{4m_e^2}{m_\phi(m_\phi - 2\varepsilon_\gamma)} \right\}^{1/2} \right] \tag{A3}$$

Inserting Eq.(A1) into Eq.(A2), we finally obtained the decay rate as

$$\begin{aligned}
\frac{d\Gamma_{br}}{d\varepsilon_\gamma} &= \frac{y^2 e^2}{(2\pi)^3} \\
&\times \left[ \left( \frac{\varepsilon_\gamma}{m_\phi} - 1 + \frac{m_\phi}{2\varepsilon_\gamma} - \frac{2m_e^2}{m_\phi\varepsilon_\gamma} + \frac{4m_e^2}{m_\phi^2} \right) \log \frac{1+\eta}{1-\eta} \right. \\
&\quad \left. + \left( \frac{m_e^2}{m_\phi\varepsilon_\gamma} + \frac{2m_e^4}{m_\phi^3\varepsilon_\gamma} \right) \frac{2\eta}{1-\eta^2} \right], \tag{A4}
\end{aligned}$$

where

$$\eta = \left( 1 - \frac{4m_e^2}{m_\phi(m_\phi - 2\varepsilon_\gamma)} \right)^{1/2}. \tag{A5}$$

In the limit  $m_\phi \gg m_e, \varepsilon_\gamma$ , we get

$$\begin{aligned}
\frac{d\Gamma_{br}}{d\varepsilon_\gamma} &\simeq \frac{y^2 e^2}{(2\pi)^3} \frac{m_\phi}{\varepsilon_\gamma} \log \frac{m_\phi}{m_e} \\
&= \Gamma_e \frac{4\alpha_{em}}{\pi} \frac{1}{\varepsilon_\gamma} \log \frac{m_\phi}{m_e}, \tag{A6}
\end{aligned}$$

where  $\Gamma_e$  is the tree level decay rate (2) or (6). This represents the familiar scaling factors, and matches to the result for annihilating particle case [5].

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